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Joint spacing: analogue and numerical simulations

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Abstract—Joint spacing distribution laws remain a controversial subject in the literature. In order to understand joint spacing, analogue and numerical modelling techniques were used. In the analogue model an evolution of the joint spacing distribution law during fracture development was observed as new fractures appear. Different processes for fracture initiation and propagation have been tested numerically including mid-point fracturing and partially or completely random processes.

This study suggests that an initiation of fractures according to a random process could explain real joint spacing distributions.

An evolution of joint spacing distribution laws from initially negative exponential to log-normal and normal is found with increasing joint development.

INTRODUCTION

BLOCK size is an important factor for engineering and petroleum geologists and is a function of the mutual orientation of intersecting sets of discontinuities (including joints, faults, bedding planes, schistosity, etc.) (I.S.R.M. 1978). The block size represents a simplified picture of the fracture network and can provide information concerning the rock mass behaviour and/or permeability. The spacing of discontinuities largely controls the size of individual blocks and therefore constitutes one of the most important block size parameters.

Since Priest & Hudson (1976), numerous papers describing the statistical or geostatistical properties of discontinuity spacing in rock have been published. Sampling methods vary from one study to another and with scale (core sampling, limited outcrops, aerial photographs, etc.), each implying a different bias. In some studies, all discontinuities (different types and different sets) are included (Hudson & Priest 1983); in others, the analysis is restricted to only one kind and/or one set of fractures (Bouroz 1990). This implies that data from such studies are not always comparable. This study considers only joints, and 'spacing' is defined as the perpendicular distance between joints of the same set.

According to most publications, the relative frequency of joint spacing can be described by different distribution laws (Fig. 1): log-normal distributions (Sen & Kazi 1984, Rouleau & Gale 1985, Bouroz 1990, Narr & Suppe 1991) or negative exponential distribution (Snow 1968, Priest & Hudson 1976, La Pointe & Hudson 1985, Villaescusa & Brown 1990). Normal spacing distributions are more rarely described (Huang & Angelier 1989).

Why one distribution occurs in one case, and another in another case, remains unexplained (Dershowitz &

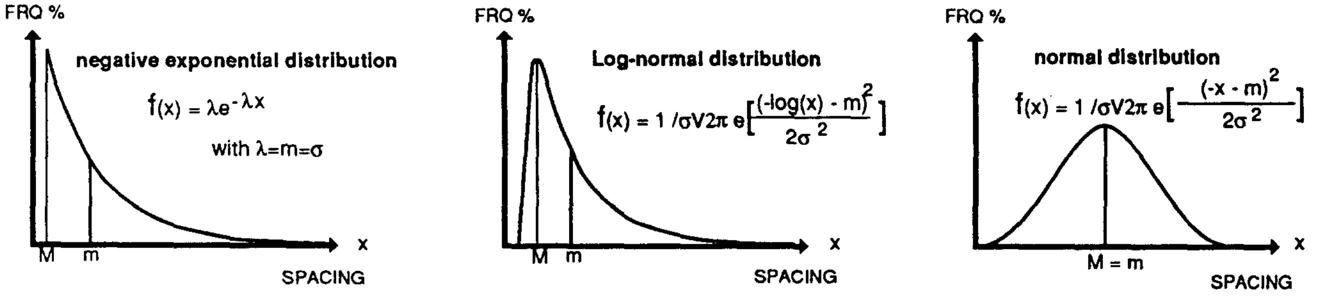

Fig. 1. The different types of distribution laws used in this paper to describe spacing distributions; M, mode; m, mean; σ, standard deviation.

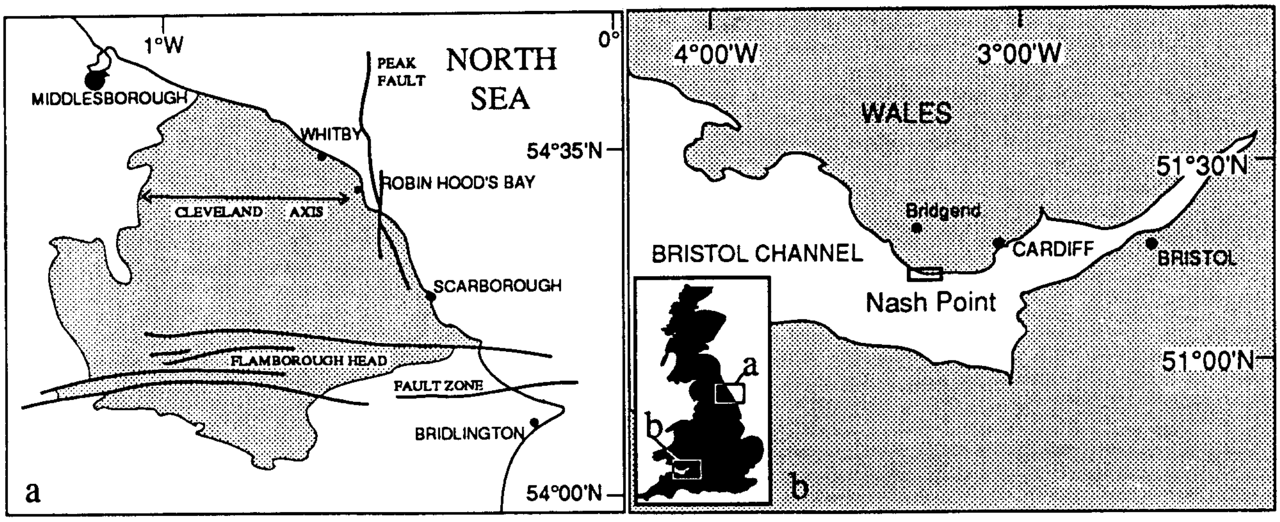

Fig. 2. Geographical situations of the studied areas. (a) Map of North Yorkshire with the Scarborough and Whitby locations. The stippled area is Jurassic outcrop. (b) Location of Nash Point in South Wales.

Einstein 1988). Sampling bias may go some way to explain the occurrence of different spacing distributions. For example in analyses based on photo-interpretation (for example Tsoutrelis *et al.* 1990) small spacings are generally ignored because of photographic resolution (Huang & Angelier 1989). Frequency distributions of this type may apparently fit with a negative exponential distribution but the distribution is truncated at inferior values. Moreover, Huang & Angelier (1989) suggest that the probability of small spacings plays a critical role in the choice of a distribution law. In field studies, another important bias could arise if large spacings are ignored (Kulatilake & Wu 1984, Pineau 1985) or, if all discontinuities (without separation of different types and different sets) are counted (for example in borehole cores), smaller spacing values are obtained. In these situations, small spacing values are represented more relative to large values. The corresponding best fit is generally a negative exponential (Priest & Hudson 1976, Barton & Zoback 1990).

Dershowitz & Einstein (1988) indicate that the different distributions may be present in unbiased data. They suggest that independently created joints can lead to negative exponential distribution, whereas joints which interact can produce log-normal distributions.

This paper aims to explain the observed variation in unbiased spacing distributions by comparing field data to the spacing distributions developed in an analogue model and numerical simulations. Different models of joint spacing development are tested. Joint spacing data are collected from the field in three areas in the U.K. The field examples allow comparison with the analogue and numerical data.

FIELD DATA

Field data are collected to provide unbiased examples of typical joint spacing distributions to constrain the numerical models. However, an interpretation of the structural significance of the joint sets in the field examples is not the aim of this work.

High quality exposures cut by a single dominant, regional joint set are selected. At each location spacing data are collected by placing a scan-line of at least 30 m perpendicular to the strike of the dominant joint set. The intersection of the joints with the scan-line is recorded. The bias commonly induced by the lack of observation of the smallest spacings is prevented because each joint is enhanced by erosion. The length of the scan-line also ensures that no high spacing values were omitted.

Scarborough and Whitby: geological settings

Joint spacing data are collected from exposures near the towns of Scarborough and Whitby, on the coast of North Yorkshire, U.K. (Fig. 2a). Middle Jurassic sediments of the Scalby Formation are exposed on the coastline 1 km to the north of Scarborough. These comprise fluvial interbedded mudstones, siltstones and thin sandstones with isolated, ribbon sandstones bodies (Alexander 1986). At Whitby shales of the Upper Lias are exposed. The main tectonic structures of the region are the Cleveland Anticline and the Peak Fault. The Cleveland Anticline is positioned between Scarborough and Whitby, and strikes approximately E–W with gentle dips. The folding is related to N–S Tertiary compression (Kent 1974, Kirby *et al.* 1987). The N–S-trending Peak Fault has downthrow to the east, with a vertical throw of between 120 and 150 m (Fox-Strangways & Barrow 1915). Analysis of recent seismic sections (Milson & Rawson 1989) and field studies (Alexander 1986) reveal that the Peak Fault forms the western border of a faulted sedimentary trough, some 30 km long and 5 km wide. The Peak Fault has been active since the Triassic, with further movements during the mid to late Jurassic (or early Cretaceous) and again during early Tertiary compression (Milson & Rawson 1989).

At both locations coastal erosion has produced extensive horizontal rock platforms on which joint traces are clearly visible. Within the Scalby Formation at Scarborough, a main joint set is present striking 165°. This set has horizontal trace lengths commonly over 50 m. The same joint set continues with small and gradual changes in direction along the coastline to the north of Scarborough to Robin Hood's Bay, where the joints strike 145° (Rawnsley *et al.* 1992, this issue; set J_2). Within the Upper Lias at Whitby the main joint set strikes approximately 180°. The joints at Robin Hood's Bay are probably the result of Tertiary compression (Rawnsley *et al.* 1992). The continuity of joint trend along the coastline strongly suggests that the joints at Scarborough and Whitby are of the same age as the joints at Robin Hood's Bay.

Nash Point: geological setting

Nash Point is located on the coast of South Wales, to the south of Bridgend (U.K. map reference SS 92 68, Fig. 2b). The rocks studied are Liassic, forming both an extensive E–W-trending cliff line and a well exposed wave cut platform at its base. Beds comprise 1 m thick hard pale grey limestone and less thick shale horizons. The dip is close to horizontal and at low tides single beds are often exposed over many hundreds of square metres. A series of conjugate strike slip faults cut the sequence trending 010° ± 10° and 160° ± 10° with horizontal trace lengths often greater than the width of exposure (100–200 m). At a distance (approximately 100 m) from the faults, the joint pattern consists of parallel and horizontally extensive joints striking 170° (Fig. 3a). In the vicinity of the faults, the joint pattern is perturbed indicating that the joints are syn- or post-faulting. The faulting and jointing are compatible with the Tertiary compression direction (Roberts 1974, Rawnsley *et al.* 1992).

Selection of probability laws

The selection of the best-fitting probability law from the spacing data is largely dependent on the number of classes into which the data is sorted. Spacing data are plotted in histograms. The significance test of chi-2 (Saporta 1990) allows one to compare observed and theoretical distributions and to determine the best fit. A very good fit is obtained with a log-normal law at Nash Point and Scarborough (Figs. 4b & c) and a negative exponential law at Whitby (Fig. 4a).

It is possible that for the same data both negative exponential and log-normal curves are obtained. A log-normal distribution is characterized by its mean, standard deviation and mode, the dominant spacing value. If the mode is close to zero, it may fall within the first class of the histogram. In this case the first class is the most represented and a negative exponential distribution provides a good fit (Fig. 4d). By increasing the number of classes until the mode value does not belong to the first class, the best fit will be a log-normal distribution (Fig. 4c). This shows that when the mode falls within the first class, the histogram seems to imply an exponential negative distribution whereas the real distribution could be log-normal.

Thus the controversy between log-normal and negative exponential laws could be due to the mode value in the log-normal laws which could be very low, and consequently the increasing part of the function can be reduced and difficult to show.

SPACING EVOLUTION DURING ANALOGUE FRACTURE SET DEVELOPMENT

Analogue method and material

In order to understand the joint development process, a series of experiments were carried out in which fracture development could be observed. Parallel fracture sets within brittle coatings have already been obtained (Durelli 1942, Rives & Petit 1990a, b, Wu & Pollard 1991). In the present study fractures have been induced in polystyrene plates subjected to traction in the presence of alcohol. The alcohol causes environmental changes in the polystyrene which facilitates polymer chain breaking (Krammer 1979). In the model, fractures result from the 'stress cracking' process in which cracks initiate at shallow defects in the polymer surface and then propagate in both directions normal to the applied negative stress (Krammer 1983, Petit & Barquins 1988). The fractures produced in the polystyrene plates are parallel, very thin and linear. The main advantages of this experimental technique is that polystyrene plates are very easy to use, the tests are prepared quickly and fractures can be seen easily with appropriate lighting.

A plate (180 × 100 × 1.5 mm) was loaded by two couples of forces (four point bending) which bend the plate with a constant radius of curvature between the two central points (cylindrical folding) (Fig. 5). The external part of the fold is thus subjected to a homogeneous extension (plane strain condition). A thin transparent plastic film covered the surface to maintain a constant alcohol ambience. Plastic tape was placed along the plate edges (Fig. 5) to inhibit fracture development at the plate boundaries by preventing alcohol from reaching the edges of the plate. Plate deformation was controlled by linear amplification of the fold. Fracture development within the plate was controlled by strain rate and time. Fractures developed in the external surface of the plate from randomly distributed defects and propagated parallel to the fold axis. The propagation of fractures was filmed for analysis. The experiments were stopped at different stages of fracturing (after the development of the first few fractures and at intermediate and high fracture densities). Spacing values along scan-lines, traced perpendicularly to fractures in the middle of the plates, were measured using a macroscope.

Joints in rocks initiate from defects (fossils, minerals, pores, etc.) and propagate away from the origin as shown by plumose structures (Helgelson & Aydin 1991) (e.g. Fig. 3b). It is assumed that in a rock mass such defects are randomly distributed. The stress-cracking technique could therefore be analogous to the development of a set of joints in an intact sedimentary layer without previous joints or faults, but containing defects.

Results

Fractures initiated from point defects randomly distributed in the plate. They propagated parallel to the fold axis (normal to the maximum tensile stress) and formed a set of parallel fractures. With increasing deformation, existing fractures propagated and new ones appeared until the density of fractures within the plate was very high and the spacing very close (Fig. 3c). Fractures stopped propagating either at the edge of the plate or when two closely spaced parallel fractures propagated toward one another and overlapped, sometimes with a 'hook' geometry (Fig. 5) similar to that described in rock (Olson & Pollard 1989). New fractures initiated at a minimum distance from the nearest fracture, approximately equal to the half of the final mean spacing. The maximum fracture density was reached when a further increase in load broke the plate.

Fracture spacing distributions are analysed at three different stages of fracture development; after the development of the first few fractures, at an intermediate fracture density and at the maximum fracture density. The results with fitted probability laws are shown in Fig. 6. The spacing distribution is negative exponential at a stage with only few fractures, log-normal at intermediate fracture density, and tends towards normal at high fracture density. It thus appears that the fracture spacing distribution law evolves with increasing fracture set development (and with increasing deformation), from a negative exponential distribution to a log-normal distribution to a quasi-normal distribution (Fig. 7). During fracture set development, the mean value decreases with increasing strain along an hyperbolic function (Fig. 8). The evolution of the mean is not linear as in the brittle coating analogue models of Wu & Pollard (1991). This difference could be partially explained by the influence of the loading time and loading rate which are not well constrained in our experiments. The discrepancy could also be linked to the much higher sensitivity of the brittle coating used by Wu and Pollard compared to that of the polystyrene plates used here (the strain gain between the first and the final fractures is 2‰ and 5%, respectively). Also the behaviour of the brittle coating is different to the polystyrene in that it contains a residual stress (Faure 1966). Nevertheless, the stress cracking experiments illustrate a possible joint development process, and the problem of the strain (or stress) which produces natural joints remains unresolved (see for example Lorenz *et al.* 1991) and is out of the scope of this paper.

NUMERICAL SIMULATION

In the last 10 years numerical simulation of rock mass fracturing has made significant advances (Miller & Borgman 1985, Massoud 1987, Chiles 1989). These simulations have been application-oriented. A fracture state is generated representing joints within a rock mass to deduce hydraulic and mechanical properties (Baecher 1983, La Pointe & Hudson 1985, Kulatilake 1988). Other numerical simulations attempt to reproduce and understand joint development processes (Narr & Suppe 1991).

Principles of simulation

It should be noted that in the numerical simulations carried out in this study only statistical and probabilistic features (mean, standard deviation, distribution laws) of joint sets are taken into consideration. The models do not directly employ mechanical parameters for joint development but the processes tested are constrained by established mechanical conditions.

The scan-line used to measure joint spacing in the field is represented numerically by an arbitrary range of values from 0 to 4000. Six hundred numbers within this range are sequentially generated, following a defined process, to represent joint intersections with the scan-line. After the generation of each 60 new 'joints', the spacing values are calculated and plotted in histograms. Probability laws are fitted to the spacing distributions. Different joint spacing development processes are tested by comparing the spacing in the model to real field data. Only the processes which generate realistic joint spacing distributions at each development step are selected. A process which generates a realistic distribution after 600 joints but unrealistic distributions at intermediate steps cannot be a viable process since the unrealistic distributions are not observed in nature. A close fitting joint spacing development process will provide information on the origin of joint periodicity. Different processes were tested to establish a possible joint development process:

Mid-point bisection process. Price (1966) suggested that in a bed subjected to extension, containing some randomly positioned pre-existing joints, the next fracture will occur in the middle of the distance corresponding to the greatest spacing (which is theoretically subjected to the greatest stress). Randomly positioned joints can result from a Poisson process, which produces a negative exponential spacing distribution (Saporta 1990). The mid-joint bisection process is tested by generating the 60 initial fractures by the Monte Carlo method (Pelletier 1971, Howard & Nolen-Hoeksema 1990) from a negative exponential distribution. The next joint is positioned by bisecting the greatest spacing (Fig. 9a). This operation is repeated until 600 joints are created.

Semi-random bisection process. This process is similar to the mid-point bisection process except that after

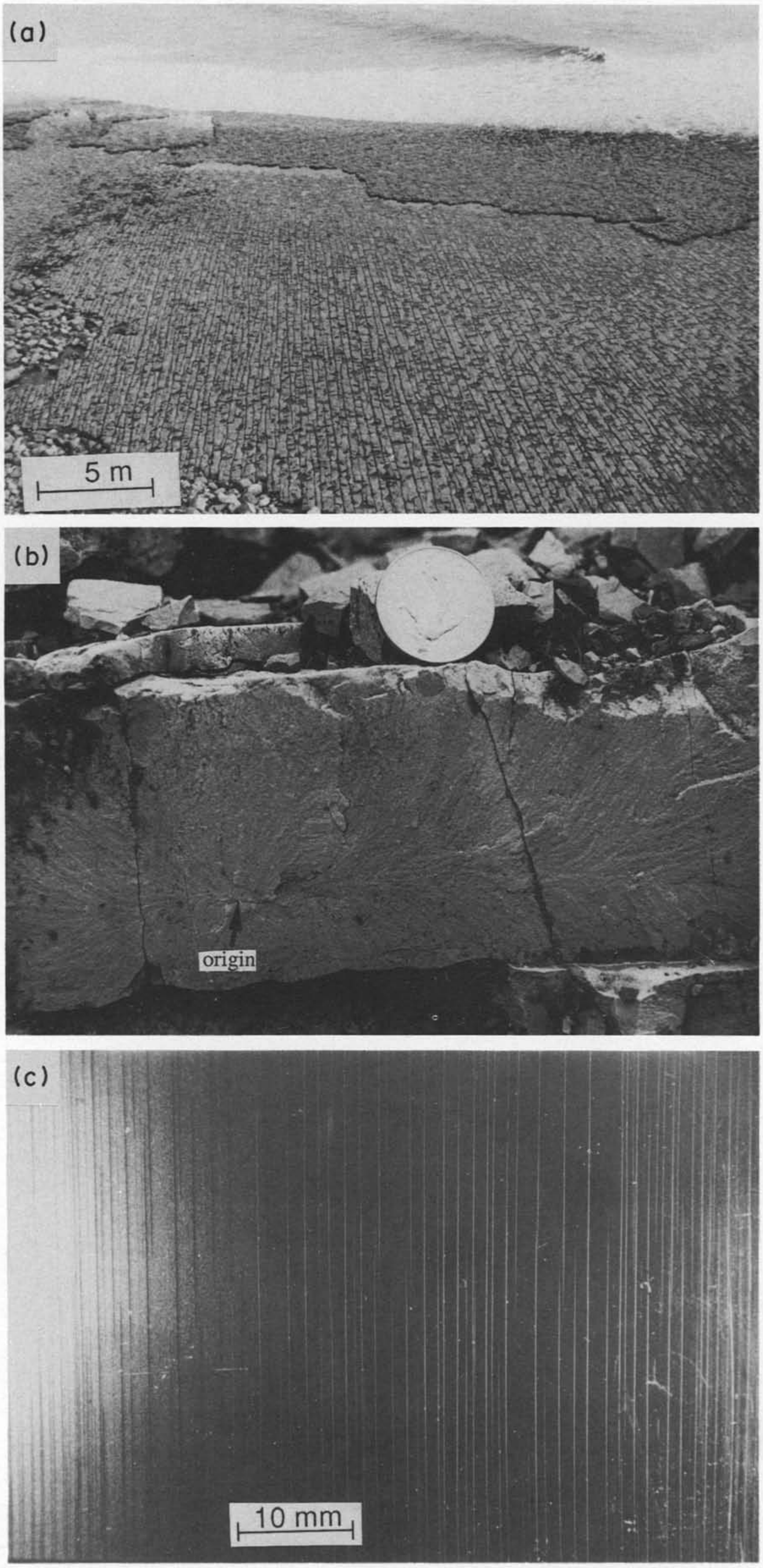

Fig. 3. (a) A joint pattern at Nash Point showing a set of linear fractures striking 170° in Liassic limestone. Average spacing is 0.3 m. Orthogonal joints rarely cross-cut the 170° joints. (b) Joint plane in a thin bed of limestone showing a plumose structure diverging from a point indicating propagation away from a point. (c) Analogue fracture set produced by the stress cracking process in a thin plate of polystyrene. The almost constant fracture spacing corresponds to a highly developed stage.

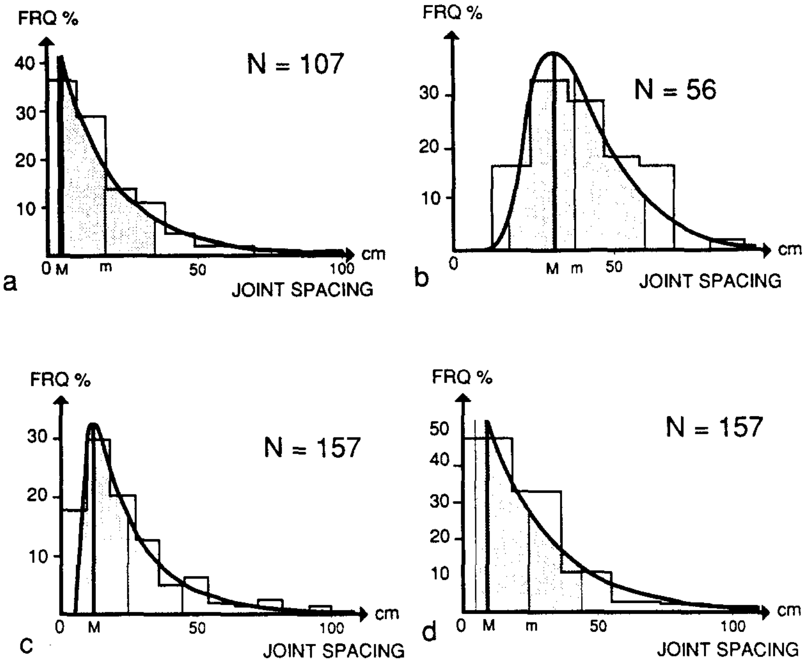

Fig. 4. Histograms and fitted distribution laws for joint spacing values sampled in three areas around the U.K.: M, mode; m, mean; *N*, number of values, dashed zones indicate standard deviations. (a) N–S joint set of Whitby (Yorkshire) with negative exponential fit. (b) 170° joint set of Nash Point (south Wales) with log-normal fit. (c) & (d) 170° joint set of Scarborough (Yorkshire); (c) log-normal fit (12 classes), (d) negative exponential fit (six classes).

generating an initial set of 60 joints with the Monte Carlo method, the greatest spacing is selected and randomly cut into two smaller spacings (Fig. 9b).

Random process. Six hundred numbers are generated independently and at random between 0 and 4000 (Fig. 9c).

Interaction process. Overlapping and 'hook' geometries in analogue modelling (Swain & Hagan 1978) and in field outcrops (Olson & Pollard 1989) are often observed and indicate the presence of interactions between fractures. The development of a fracture produces stress relaxation in its vicinity (Pollard *et al.* 1982, Segall & Pollard 1983). This relaxation affects a small

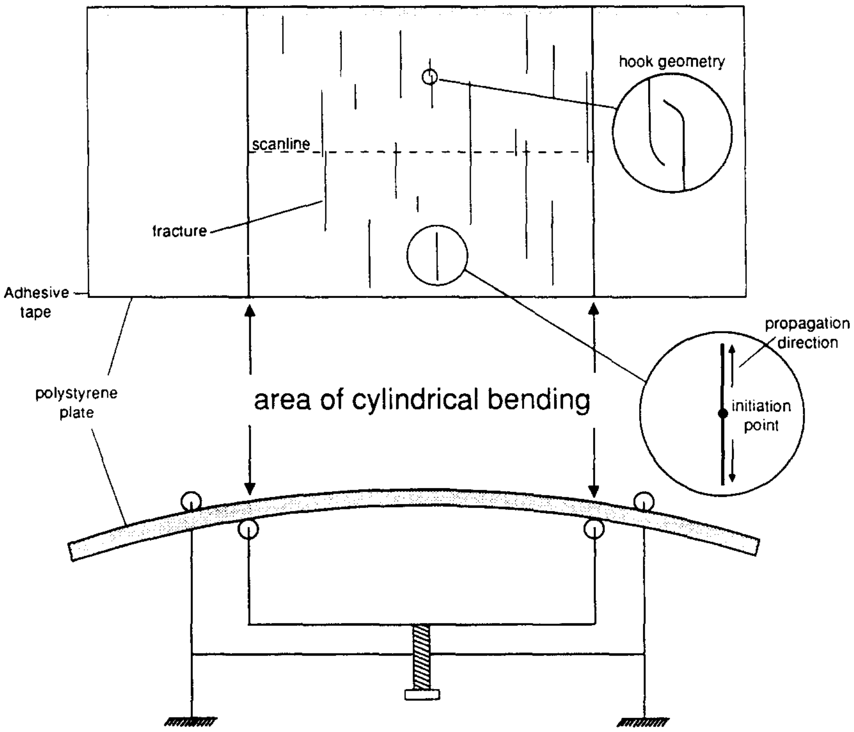

Fig. 5. Sketch of the analogue apparatus and representation of a few fractures in the external part of the polystyrene plate. Fractures propagate away from initiation points randomly distributed in the plate and stop either at the edges of the plate or when they interact with one another forming hook geometries. See also Fig. 3(c).

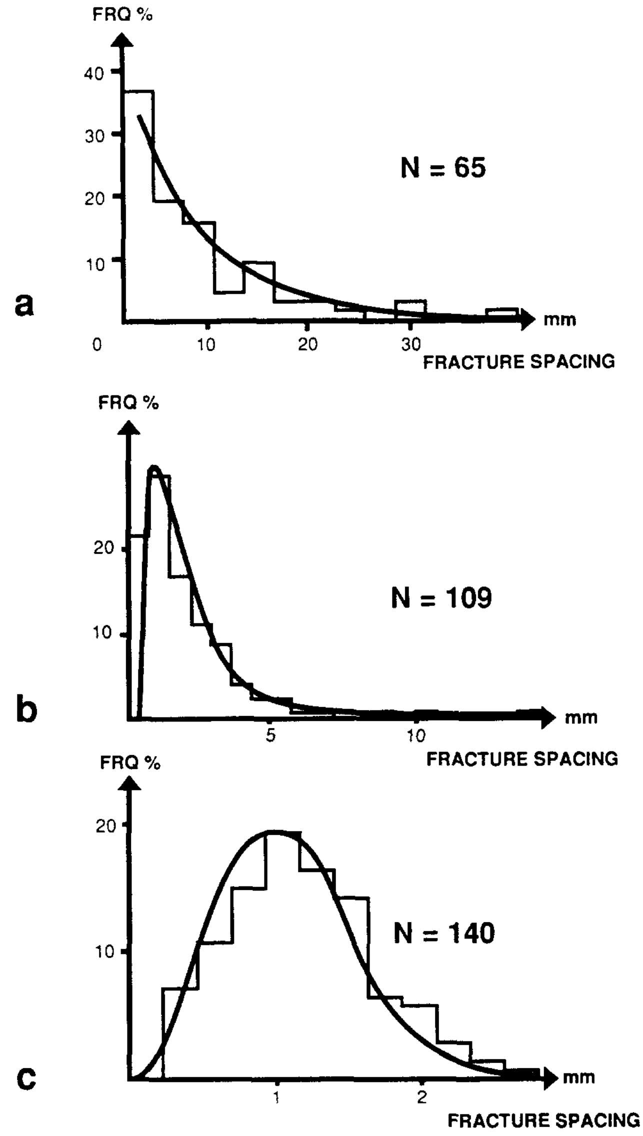

Fig. 6. Histograms and fitted distribution laws of the spacing of the generated fractures in the analogues model. (a) After the few first fractures, (b) intermediate fracture density and (c) high fracture density.

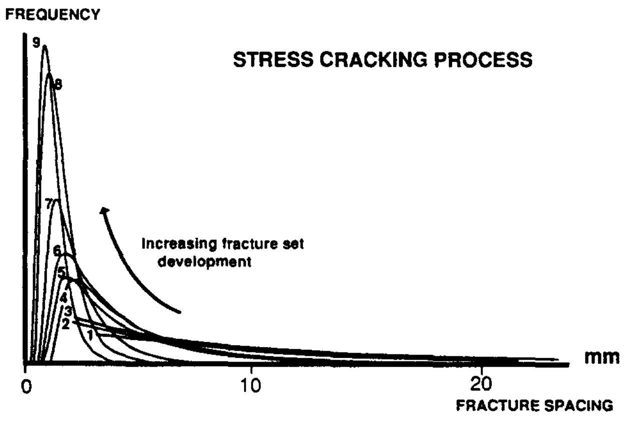

Fig. 7. Fitted distribution laws of the measured spacing from the generated fractures in the analogue model. Nine developmental stages are represented on the same horizontal axis with the same area beneath each curve. Numbers indicate increasing development stages.

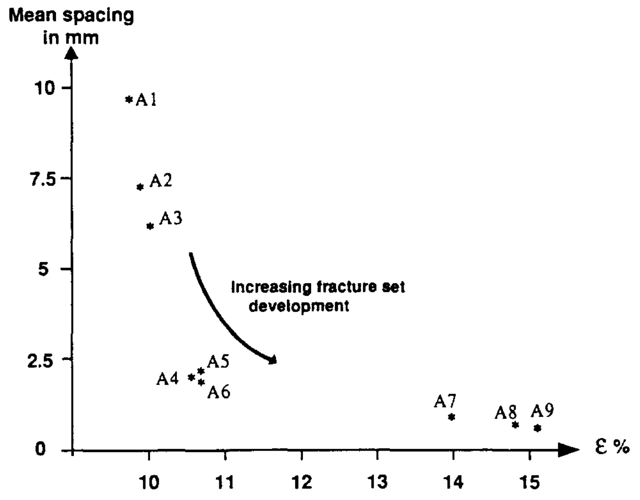

Fig. 8. Mean value vs % strain in the plates. The decrease of the mean value during fracture set development is not linear (see text).

zone around each fracture in which no new fracture can form. The quantification of this interaction zone remains unknown but seems to be linked to the mechanical rock properties (Pollard & Aydin 1988) and the bed thickness in sedimentary layers (Souffaché & Angelier 1989). To take into account this interaction in simulations, the random process is modified to include shadow zones around each fracture (Fig. 9d).

Results

The spacing distribution functions obtained in the mid-point bisection process (Fig. 10) do not fit any theoretical classical unimodal laws. Generated spacing values are rarely small, producing distributions with a negative asymmetry. This process cannot produce realistic joint spacing distributions, and consequently is probably not representative of a real joint development

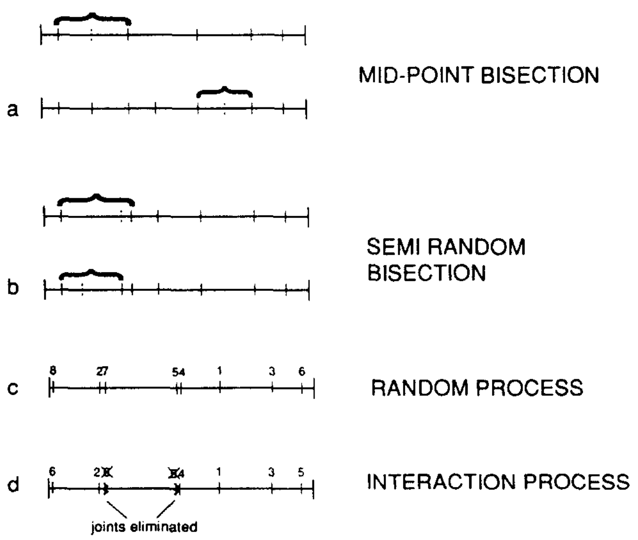

Fig. 9. Sketches illustrating the different numerical processes tested: (a) mid-point bisection, the greatest spacing is selected then cut in the middle; (b) semi-random bisection, the greatest spacing selected then randomly cut; (c) random process, each fracture is generated at random; (d) interaction process, each fracture is generated at random but eliminated if its position is too close to another one.

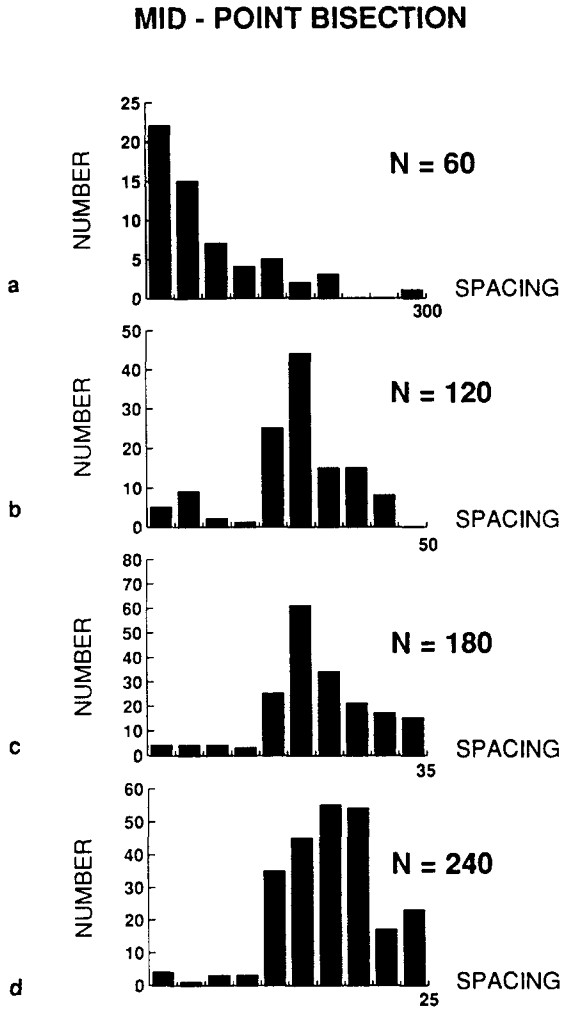

Fig. 10. Histograms showing the spacing distributions of generated fractures with the mid-point bisection process. (a) Spacing distribution of the 60 pre-existing fractures assumed to be negative exponential; (b)–(d) spacing distributions at each stage of 60 new fractures following the mid-point bisection process.

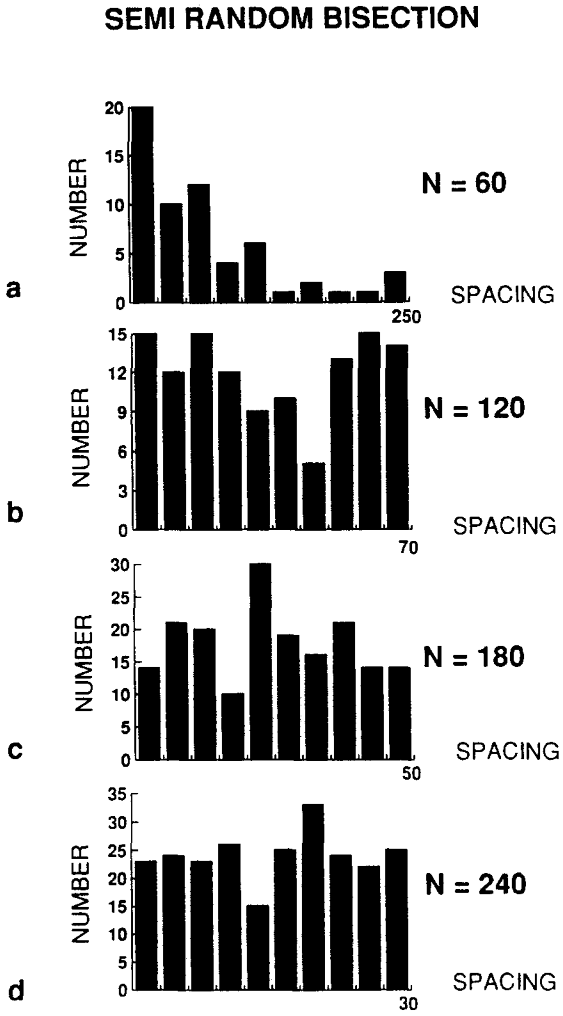

Fig. 11. Histograms showing the spacing distributions of generated fractures with the semi-random bisection process. (a) Spacing distribution of the 60 pre-existing fractures assumed to be negative exponential; (b)–(d) spacing distributions at each stage of 60 new fractures following the semi-random bisection process. The obtained distributions are uniform.

process. This conclusion is in agreement with the mechanical model of Narr & Suppe (1991).

The distributions obtained in the semi-random bisection process after doubling the number of joints (Fig. 11) show a uniform tendency (i.e. values are equally probable); large spacings quickly disappear and generated spacings are uniformly distributed between zero and the greatest spacing value. This process does not give distributions comparable to those of real situations. Thus, it is unlikely that this process is representative of a realistic joint development process.

In the random process, as in the simulations described above, the random generation of the first 60 fractures creates a negative exponential spacing distribution with the mean almost equal to the standard deviation. Random positioning of 60 new fractures (independently of the first fractures) leads to a new negative exponential spacing distribution with a smaller mean but always close to the standard deviation (Fig. 12). The random process allows simulation of the observed evolution of the analogue mean spacing with development. The random position of each new fracture generated is more probable in regions of larger spacings than small spacing (the cumulative probability is greater). This implies that great spacings become fewer during this process and the number of small spacings increases continuously. This process permits simulation of the evolution of the negative exponential distribution function but cannot produce log-normal law.

Spacing distributions in the interaction process (Fig. 13) are found to be negative exponential during the first steps, then they become log-normal and finally are almost normal after 600 fractures are generated. The standard deviation remains close to the mean value during the first stages and then after further stages the standard deviation decreases continuously to one-third of the mean value. This decrease indicates that this process is not a Poisson process but results from more than one parameter.

The interaction process provides a realistic joint spacing distribution (exponential or log-normal or normal) at each stage of joint development. The evolution of the spacing distribution in the interaction process is also in close agreement with the evolution shown in the

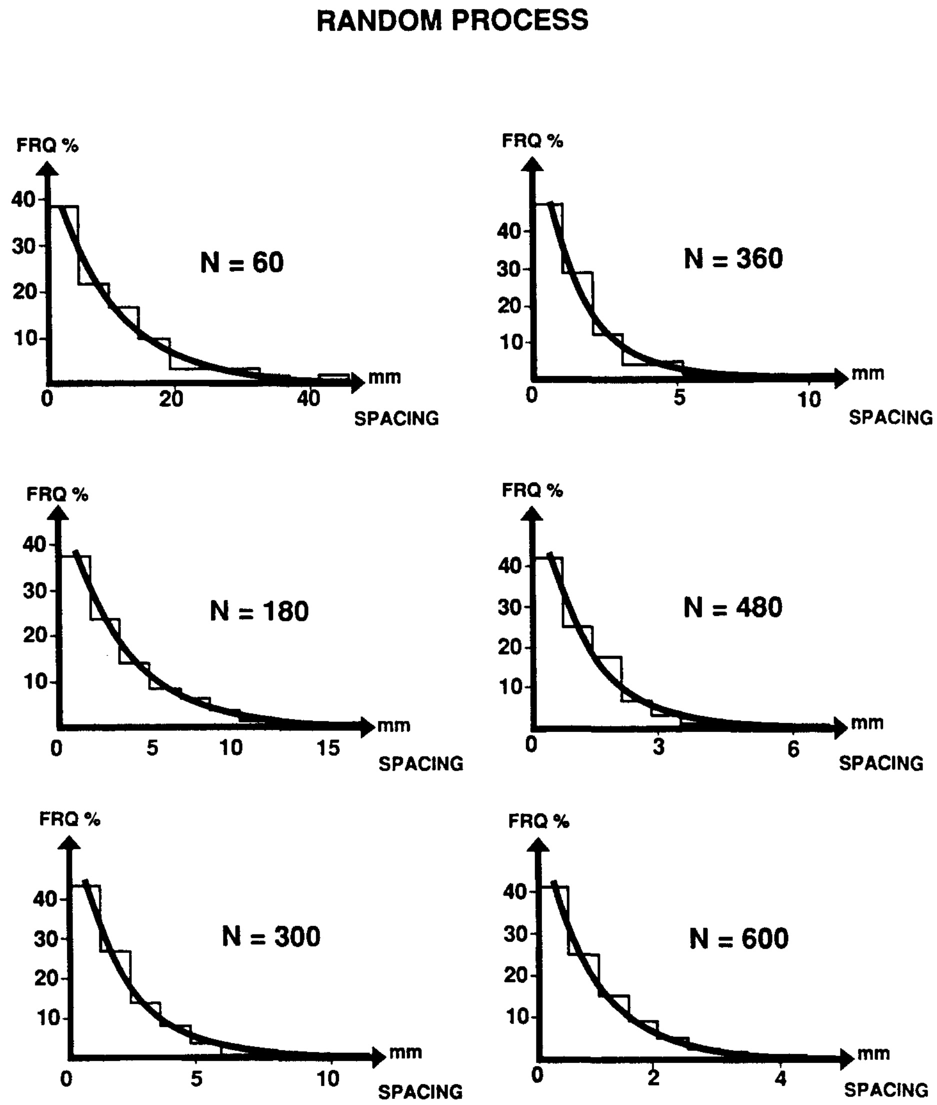

Fig. 12. Histograms plus fitted distribution laws of the spacing distributions of the generated fractures, at different steps, with the random process. With increasing fracture density (increasing *N*) the type of the distribution law stays negative exponential.

analogue model (Fig. 14). The evolution from a negative exponential distribution to a normal distribution suggests that at the beginning, fracture spacing is governed by a random process; then with increasing fracture density, the shadow zones are more and more numerous and their importance becomes greater until the final fractures have no freedom, and a normal distribution is reached corresponding to a near constant close spacing.

It seems that the spacing distribution can be considered as an indicator of joint set development stage. The evolution of the mode/mean ratio (from 0 to 1) with increasing number of generated fractures could also indicate the saturation level of a set. The evolution of the mode/mean ratio in the numerical model (Fig. 15a) is in agreement with the evolution stages in the analogue model (Fig. 15b). A low value (towards 0) signifies that the set is poorly developed; a high value (towards 1) indicates a nearly saturated set.

DISCUSSION

In the model of joint spacing development suggested by the analogue experiment, spacing periodicity results from an 'interaction model' in which individual fractures nucleate at various randomly distributed points throughout a layer at various times. Fractures propagate from these points in both directions perpendicular to the traction direction. No fracture can nucleate at less than a certain limiting distance (the interaction zone) from any other fracture. The process continues until all available space is occupied and the joint set is 'saturated'. Thus the saturated joint spacing has a limited range of values (Cobbold 1979). Numerical results also show that a completely random process with a small shadow zone around each fracture can simulate joint spacing development (interaction process). The interaction models (analogue and numerical) simulate joint development in

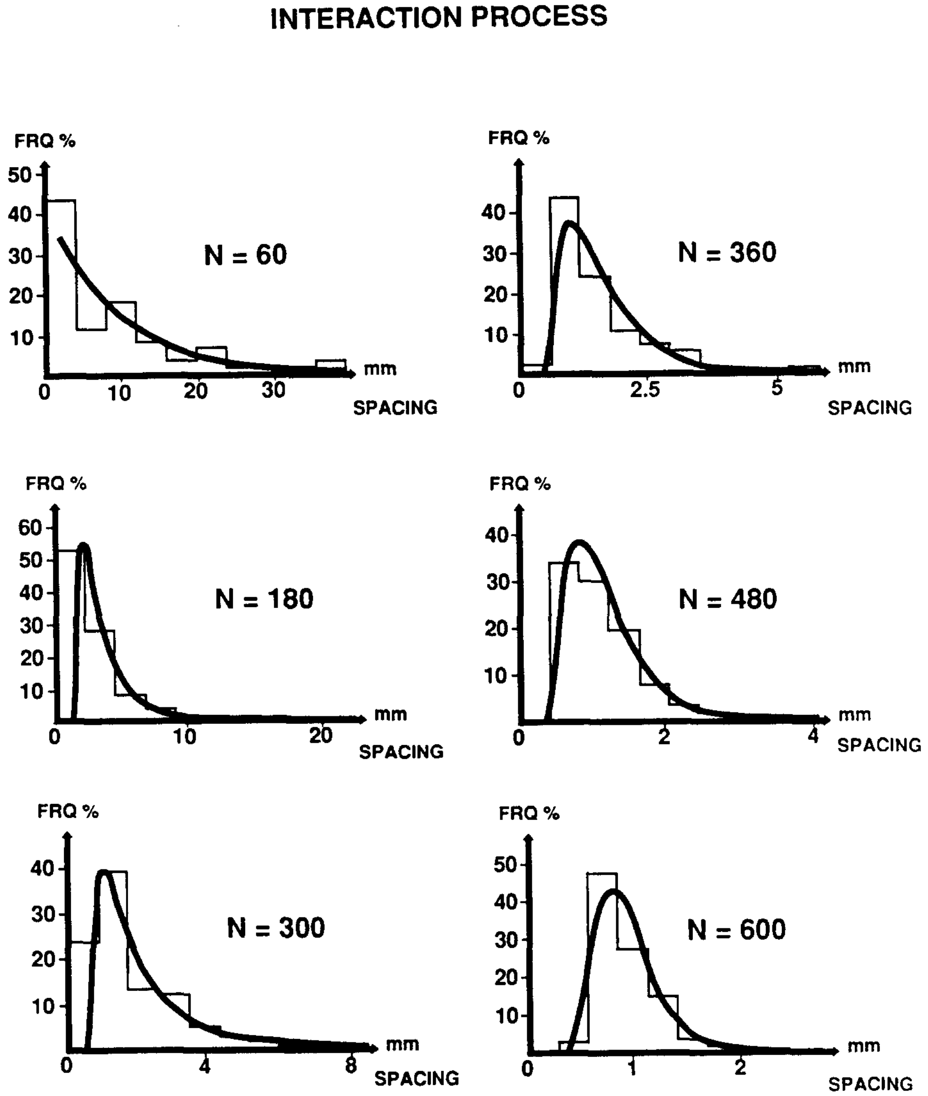

Fig. 13. Histograms plus fitted distribution laws of the spacing distributions of the generated fractures, at different steps, with the interacting process. With increasing fracture density (increasing N) the type of the distribution law varies from negative exponential to log-normal to almost normal after 600 fractures.

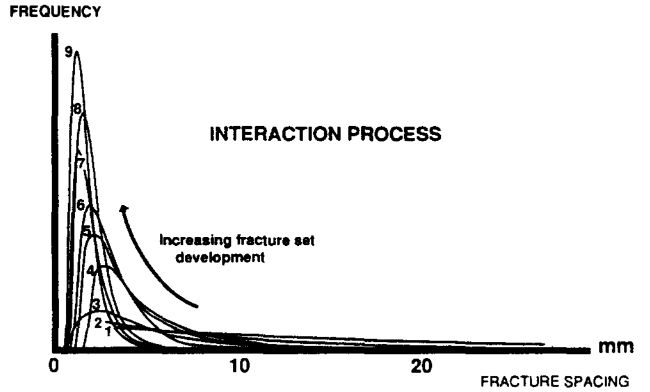

Fig. 14. Fitted distribution laws of the measured spacing from the generated fractures with the numerical interaction process. Different stages of development are represented on the same horizontal axis and with the same area beneath each curve. Numbers indicate increasing fracture set development. The diagram shows the evolution of the distribution laws with increasing fracture saturation.

a sedimentary layer because in both cases fractures initiate in tension from randomly distributed defects and then propagate away from these points in both directions.

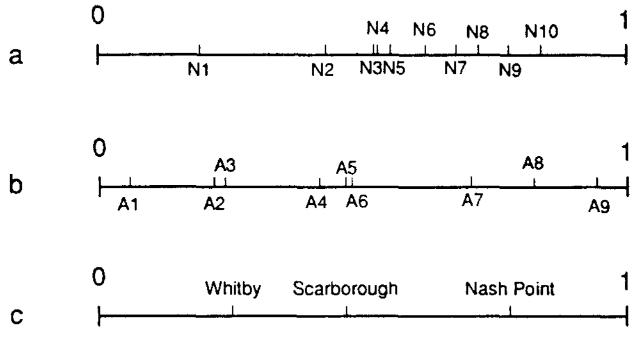

Fig. 15. Mode/mean ratio: (a) from numerical data, (b) from analogue data and (c) from field data. Increasing numbers in (a) and (b) indicate more developed stages. The position of the ratio for field data could indicate the saturation level of the joint set.

The interaction model implies that the criteria for initiation and propagation of the joint are different. In the model a joint can initiate only from one of the randomly distributed defects that lies outside the interaction zones around existing joints. The area available for joint initiation decreases with the cumulative length of the joints. The cumulative length of the joints is related to both the number of joints and the fracture length distribution. Once initiated, a joint will propagate until it enters the interaction zone of another joint, or the driving stress becomes insufficient for continued propagation. Because the joints are co-planar, the only interaction between them is 'end to end' creating echelon or hook geometries (Olson & Pollard 1989). A propagating joint only 'sees' the ends of the other joints and the widths of the interactions zones that surround them. A joint will only terminate if it enters the 'width' of rock occupied by the interaction zone of another joint. The width of the interaction zone varies only slightly for different joints and therefore the likelihood of a joint entering the interaction zone of another joint increases only with increasing number of joints. The number of joints and the cumulative length of the joints increases with the evolution of the joint set.

The model suggests that joint set development is controlled by the spatial distribution of initiation points, the size of the interaction zones, the initiation and propagation criteria of the joint set, and the stage of evolution. As a result, the joint spacing distribution is related to the fracture length distribution.

The evolution of spacing distribution type with joint set development indicates that for each set of joints at a given location, the spacing distribution depends upon the stage of joint set development. The final stage of development corresponds to the 'saturation level' of Narr & Suppe (1991). Consequently, the type of law fitted could indicate the degree of evolution in the development of a set. For the generated spacing distributions the type of law is represented by the mode/mean ratio which could also indicate the stage of development of a real joint set. Assuming this ratio is valid for the joint spacings measured in the field, the spacing distributions could indicate the saturation level of each set. Increasing values are obtained for Whitby, Scarborough and Nash Point (Fig. 15c). The joint set at Whitby is less developed compared to the same set 30 km to the south in Scarborough. This difference for the same regional joint set can be explained by the lithology variation between the two areas, mudstone at Whitby and sandstone bodies at Scarborough. The joint set at Nash Point appears to have reached a quasi-saturated development level.

The spacing distribution laws, negative exponential, log-normal and normal, produced in the analogue model have a variation coefficient (mean/standard deviation) between 1 and 3. As a consequence, during static simulation of joint spacing distribution the chosen laws must be within this range.

The mechanical process producing the classical relationship between joint spacing and bed thickness (Novikova 1947, Hobbs 1967, Ladeira & Price 1981) remains unclear despite recent work (Angelier *et al.* 1989, Souffaché & Angelier 1989). An important consequence of the evolution of the spacing distribution is that each bed in a series of beds can have reached different developmental levels. Consequently a measured mean joint spacing value is not always the minimum 'potential' mean spacing of a layer, and the classical relationship between bed thickness and mean spacing is doubtful. We suggest that the valid relationship between joint spacing and bed thickness is more complicated than the previously suggested average spacing/bed thickness correlations, because the size of the interaction zone and the stage of joint set evolution must be considered. The interaction zone seems to be related to bed thickness and mechanical properties, although further work is required to quantify this feature. A further complication to the development of joint spacing may result from vertical propagation of joints between beds (Helgeson & Aydin 1991) and variations in loading conditions (Dyer 1988, Olson & Pollard 1989, Wu & Pollard 1991).

The interaction model implies that the origin of joint spacing distribution cannot be considered as a one-dimensional problem as previously suggested (Price 1966, Hobbs 1967, Narr & Suppe 1991) but instead must be treated as a three-dimensional problem. However, provided joints initiate from randomly distributed points they will also intersect a one-dimensional scan-line at random. The use of a scan-line to describe joint spacing is therefore valid. The interaction process also suggests that it is impossible to predict the position of future fractures relative to existing ones.

Some authors propose more sophisticated fitted laws including gamma (Huang & Angelier 1989) or Weibull (Bardsley *et al.* 1990) to describe joint spacing distributions. These laws vary with two parameters and include both the basic distributions (log-normal and negative exponential). Because of the control of two parameters, these laws can provide the best fits to data sets and could describe the evolution of joint spacing distributions; however, further work is required to constrain the two possible parameters. Further developments of this study will include dynamic joint development simulations taking into account real joint length distributions (Razack 1982).

The present work concerns a very idealized situation with only one joint set, without any interaction with other structures (e.g. faults, other joint sets), but it serves to underline the value of individual analyses of joint sets.

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